Search for new physics indirect effects in $e^+e^- \to W^+W^-$ at linear colliders with polarized beams

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Abstract

We discuss the potential of a $0.5\,TeV$ linear collider to explore manifestations of extended (or alternative) electroweak models of current interest, through measurements of the reaction $e^+e^- \to W^+W^-$ with both initial and final states polarization. Specifically, we consider the possibility to put stringent constraints on lepton mixing (or extended lepton couplings) and $Z-Z'$ mixing, showing in particular the usefulness of polarization in order to disentangle these effects.

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It is generally believed that, although tremendously successful from the phenomenological point of view, the Standard Model (SM) should not be considered as the ultimate theory. Most proposed schemes attempting to address the conceptual problems of the SM predict the existence of additional building blocks, namely new heavy gauge bosons and fermions. These extra degrees of freedom might be too heavy for direct production and, in high energy reactions, they could manifest themselves only as "indirect" effects induced by mixings with the conventional SM fermions and bosons. In general, such mixing effects reflect the underlying extended gauge symmetry and/or the Higgs sector of the model.

Studies of high energy e^+e^- annihilation can give an opportunity to make precise tests of mixing effects. In $e^+e^- \to \bar{f}f$, one can search for indirect effects at the standard Z resonance [1]-[7]. At the higher energies of planned linear colliders, this reaction should be convenient mostly for the direct search of these new matter particles [8]-[10], if their production is allowed.

Conversely, at linear colliders, the process

$$e^+ + e^- \to W^+ + W^-, \tag{1}$$

should be quite sensitive to indirect new physics effects [11, 12], which can destroy the SM gauge cancellation among the different contributions, and hence cause deviations of the cross section from the SM prediction which can increase with the CM energy. In fact, considering the cross sections for longitudinally polarized initial e^-e^+ beams, namely σ^{LR} and σ^{RL}, significant improvements of the present limits on lepton mixing and Z-Z' mixing could be obtained. In this paper, we would like to extend the analyses of lepton and Z-Z' mixing presented in [11, 12], and discuss the role of final W^+W^- polarization measurements (combined with initial state polarization) in order to further improve the bounds. In addition, we shall see that combining initial and final states polarization provides a simple way to derive separate bounds for lepton mixing and Z-Z' mixing.

Although the discussion could be done quite in general, we believe it more interesting and useful to present this kind of analysis, and quantitatively derive the bounds, making reference to some specific extended (or alternative) electroweak models of current interest.

Specifically, for the leptons we shall limit ourselves to the cases listed in Table 1, where the considered new fermions are either doublets or singlets under the gauge symmetry $SU(2)$. We also assume that the new, "exotic" fermions only mix with the standard ones within the same family (the electron and its neutrino being the ones relevant to process (1)), which assures the absence of tree-level generation changing neutral currents. The needed fermion mixing formalism has been introduced, e.g., in [12]. Basically, denoting by ν, e, N and E the mass eigenstates, the neutral current couplings of leptons to Z and Z' can be written, respectively, as

$$g_a^e = g_a^{e^0} c_{1a}^2 + g_a^{E^0} s_{1a}^2; \qquad\qquad g_a'^e = g_a'^{e^0} c_{1a}^2 + g_a'^{E^0} s_{1a}^2, \tag{2}$$

where $f^0 \equiv e^0$, E^0 are gauge-eigenstates, and $a = L, R$. Moreover, in Eq. (2) $c_{1a} = \cos\psi_{1a}$ and $s_{1a} = \sin\psi_{1a}$, with ψ_{1a} the mixing angle between the two charged leptons.

The charged current couplings are given by:

$$
\begin{aligned}
G_L^\nu &= c_{1L}c_{2L} - 2T_{3L}^E s_{1L}s_{2L}; & G_R^\nu &= -2T_{3R}^E s_{1R}s_{2R}, \\
G_L^N &= -s_{2L}c_{1L} - 2T_{3L}^E c_{2L}s_{1L}; & G_R^N &= -2T_{3R}^E c_{2R}s_{1R},
\end{aligned} \tag{3}
$$

and, analogously to (2), $c_{2a} = \cos\psi_{2a}$ and $s_{2a} = \sin\psi_{2a}$ refer to the mixing between the neutral leptons.

In turn, Z-Z' mixing is introduced through the relation

$$
\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z \\ Z' \end{pmatrix}, \tag{4}
$$

where Z, Z' are weak-eigenstates, Z_1, Z_2 are mass-eigenstates and ϕ is the Z-Z' mixing angle.

As a class of models where lepton mixing and Z-Z' mixing can be simultaneously present, we will consider in the sequel the case of E_6 models. In these extended schemes, the fermion couplings to Z are the familiar SM ones:

$$
g_a^{f^0} = \left(T_{3a}^{f^0} - Q_{em,a}^{f^0} s_W^2 \right) g_Z, \tag{5}
$$

with $s_W^2 = \sin^2\theta_W$ and $g_Z = 1/s_W c_W$, and the couplings to Z' are:

$$
\begin{aligned}
g_L^{\prime e^0} &= (3A + B) g_{Z'}; & g_R^{\prime e^0} &= (A - B) g_{Z'} \\
g_L^{\prime E^0} &= -(2A + 2B) g_{Z'}; & g_R^{\prime E^0} &= (-2A + 2B) g_{Z'}.
\end{aligned} \tag{6}
$$

In Eq. (6): $g_{Z'} = g_Z s_W$, $A = \cos\beta/2\sqrt{6}$, $B = \sqrt{10}\sin\beta/12$, with β specifying the orientation of the $U(1)'$ generator in the E_6 group space [13]. The most commonly considered models are Z'_χ, Z'_ψ and $-Z'_\eta$ models, which are specified by $\beta = 0, \pi/2$ and $\left(\pi - arctan\sqrt{5/3}\right)$, respectively.

Finally, taking Eq. (4) into account, the leptons neutral current couplings to Z_1 and Z_2 are, respectively:

$$
g_{1a}^f = g_a^f \cos\phi + g_a^{\prime f} \sin\phi; \qquad g_{2a}^f = -g_a^f \sin\phi + g_a^{\prime f} \cos\phi. \tag{7}
$$

Leptons	Vector doublets	Vector singlets	Mirror fermions
$SU(2)$	$\begin{pmatrix} N^0 \\ E^0 \end{pmatrix}_L$	N_L^0, N_R^0	N_L^0, E_L^0
structure	$\begin{pmatrix} N^0 \\ E^0 \end{pmatrix}_R$	E_L^0, E_R^0	$\begin{pmatrix} N^0 \\ E^0 \end{pmatrix}_R$

Table 1: $SU(2)$ assignments for new leptons, E and N refer to electric charge -1 and 0 respectively. The superscript "0" means weak eigenstates.

Obviously, the SM case is reobtained when all (fermion and gauge boson) mixing angles are put equal to zero.

The cross section of process (1) can be expressed in general as

$$
\frac{d\sigma(P_1, P_2)}{d\cos\theta} = \frac{1}{4}\left[(1+P_1)\cdot(1-P_2)\frac{d\sigma^{RL}}{d\cos\theta} + (1-P_1)\cdot(1+P_2)\frac{d\sigma^{LR}}{d\cos\theta}\right.
$$
$$
\left. + (1+P_1)\cdot(1+P_2)\frac{d\sigma^{RR}}{d\cos\theta} + (1-P_1)\cdot(1-P_2)\frac{d\sigma^{LL}}{d\cos\theta}\right], \quad (8)
$$

where P_1 (P_2) are less than unity, and represent the actual degrees of longitudinal polarization of e^- (e^+).

The relevant polarized differential cross sections for $e_a^- e_b^+ \to W_\alpha^- W_\beta^+$ can be written as

$$
\frac{d\sigma_{\alpha\beta}^{ab}}{d\cos\theta} = C\cdot\sum_{k=0}^{k=2} F_k^{ab}\,\mathcal{O}_{k\,\alpha\beta}. \quad (9)
$$

Here $C = \pi\alpha_{e.m.}^2\beta_W/2s$, with $\beta_W = (1-4M_W^2/s)^{1/2}$ the W velocity in the CM frame, and the helicities of the initial e^-e^+ and final W^-W^+ states are labeled as $ab = (RL, LR, LL, RR)$ and $\alpha\beta = (LL, TT, TL)$, respectively. The $\mathcal{O}_k$ are functions of the kinematical variables which characterize the various possibilities for the final W^+W^- polarizations ($TT, LL, TL+LT$ or the sum over all W^+W^- polarization states for unpolarized W's). The F_k are combinations of coupling constants including the two kinds of mixings.

For the LR case we have

$$
F_0^{LR} = \frac{1}{16s_W^4}\left[(G_L^\nu)^2 + r_N\left(G_L^N\right)^2\right]^2,
$$
$$
F_1^{LR} = 2\left[1 - g_{WWZ_1}g_{1L}^e\cdot\chi_1 - g_{WWZ_2}g_{2L}^e\cdot\chi_2\right]^2,
$$
$$
F_2^{LR} = -\frac{1}{2s_W^2}\left[(G_L^\nu)^2 + r_N\left(G_L^N\right)^2\right]\left[1 - g_{WWZ_1}g_{1L}^e\cdot\chi_1 - g_{WWZ_2}g_{2L}^e\cdot\chi_2\right], (10)
$$

where the χ_j are the Z_1 and Z_2 propagators, i.e. $\chi_j = s/(s-M_j^2+iM_j\Gamma_j)$. Also, $r_N = t/(t-m_N^2)$, with $t = M_W^2 - s/2 + s\cos\theta\,\beta_W/2$, and m_N is the neutral heavy lepton mass. The term proportional to r_N represents the neutral heavy lepton exchange diagram in the t-channel, which accompanies the ν-exchange. Finally, in Eq. (10), $g_{WWZ_1} = \cot\theta_W\cos\phi$ and $g_{WWZ_2} = -\cot\theta_W\sin\phi$.

The RL case is simply obtained from Eq. (10) by exchanging $L \leftrightarrow R$. Also, for the LL and RR cases there is only N-exchange:

$$
F_0^{LL} = F_0^{RR} = \frac{1}{16s_W^4}r_N^2\left(G_L^N G_R^N\right)^2. \quad (11)
$$

Eq. (10) is obtained in the approximation where the imaginary parts of the Z_1 and Z_2 boson propagators are neglected. Accounting for this effect requires the replacements $\chi_j \to Re\chi_j$ and $\chi_j^2 \to |\chi_j|^2$ ($j = 1, 2$) in the right-hand side of Eq. (10).

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Concerning the $\mathcal{O}_{k\,\alpha\beta}$ in Eq. (9), for the cross section $\dfrac{d\sigma(e^-e^+ \to W_L^- W_L^+)}{d\cos\theta}$ with any initial polarization, we have (with $|\vec{p}| = \sqrt{s}\beta_W/2$):

$$\mathcal{O}_{0,LL} = \frac{s\sin^2\theta}{4t^2 M_W^4}\left[s^3(1+\cos^2\theta) - 4M_W^4(3s+4M_W^2) - 4(s+2M_W^2)|\vec{p}|s\sqrt{s}\cos\theta\right],$$

$$\mathcal{O}_{1,LL} = \frac{s^3 - 12sM_W^4 - 16M_W^6}{8sM_W^4}\sin^2\theta,$$

$$\mathcal{O}_{2,LL} = \frac{1-\cos^2\theta}{t}\left[\frac{|\vec{p}|s\sqrt{s}(s+2M_W^2)}{2M_W^4}\cos\theta - \frac{s^3 - 12sM_W^4 - 16M_W^6}{4M_W^4}\right]. \tag{12}$$

For the transverse (TT) cross section $\dfrac{d\sigma(e^+e^- \to W_T^+ W_T^-)}{d\cos\theta}$ we have:

$$\mathcal{O}_{0,TT} = \frac{4s}{t^2}\left[s(1+\cos^2\theta) - 2M_W^2 - 2|\vec{p}|\sqrt{s}\cos\theta\right]\sin^2\theta,$$

$$\mathcal{O}_{1,TT} = \frac{4|\vec{p}|^2}{s}\sin^2\theta,$$

$$\mathcal{O}_{2,TT} = \frac{\sin^2\theta}{t}\left[4|\vec{p}|\sqrt{s}\cos\theta - 8|\vec{p}|^2\right]. \tag{13}$$

For the production of one longitudinal plus one transverse vector boson $(TL + LT)$ we have:

$$\mathcal{O}_{0,TL} = \frac{2s}{t^2 M_W^2}\left[s^2(1+\cos^4\theta) - 4|\vec{p}|\sqrt{s}\,\cos\theta(4|\vec{p}|^2 + s\cos^2\theta)+\right.$$
$$\left. 4M_W^4(1+\cos^2\theta) + 2s(s-6M_W^2)\cos^2\theta - 4sM_W^2\right],$$

$$\mathcal{O}_{1,TL} = \frac{4|\vec{p}|^2}{M_W^2}(1+\cos^2\theta),$$

$$\mathcal{O}_{2,TL} = \frac{4|\vec{p}|\sqrt{s}}{tM_W^2}\left[(4|\vec{p}|^2 + s\cos^2\theta)\cos\theta - 2|\vec{p}|\sqrt{s}(1+\cos^2\theta)\right]. \tag{14}$$

Finally, in the case of equal helicities of initial electron and positron beams (LL and RR):

$$\mathcal{O}_{0,LL} = 4\frac{s\,m_N^2}{M_W^4},$$

$$\mathcal{O}_{0,TT} = 8\frac{s\,m_N^2}{t^2}(1+\cos^2\theta),$$

$$\mathcal{O}_{0,TL} = 2(1+\beta_W^2)\left(\frac{m_N}{M_W}\frac{s}{t}\right)^2\sin^2\theta. \tag{15}$$

In order to assess the sensitivity of the different cross sections to ordinary lepton-exotic lepton mixing and to Z-Z' mixing, we introduce the deviation from the SM

cross section $\Delta\sigma = \sigma - \sigma_{SM}$, where $\sigma \equiv \sigma(z_1, z_2) = \int_{z_1}^{z_2} (d\sigma/dz)\, dz$ $(z = \cos\theta)$, and z_1, z_2 specify the kinematical range experimentally allowed. Then, we define a χ^2 function

$$\chi^2 = \left(\frac{\Delta\sigma}{\delta\sigma_{SM}}\right)^2, \tag{16}$$

with $\delta\sigma_{SM}$ the accuracy experimentally obtainable on $\sigma(z_1, z_2)_{SM}$. Including both statistical and systematical errors, $\delta\sigma_{SM} = \sqrt{(\delta\sigma_{stat})^2 + (\delta\sigma_{syst})^2}$, where $(\delta\sigma/\sigma)_{stat} = 1/\sqrt{L_{int}\varepsilon_W\sigma_{SM}}$ with L_{int} the integrated luminosity and ε_W the efficiency for W^+W^- reconstruction in the considered polarization state. For that we take the channel of lepton pairs $(e\nu + \mu\nu)$ plus two hadronic jets, which corresponds to $\varepsilon_W \simeq 0.3$ [14]-[17].[3] The criterion we shall follow to derive bounds on the mixing angles will be to impose that $\chi^2 \le \chi^2_{crit}$, where χ^2_{crit} is a number which specifies a chosen confidence level and in principle can depend on the details of the analysis.

For simplicity, we start our analysis by considering the case where there is lepton mixing only, and no Z-Z' mixing. Present limits on s_1^2 and s_2^2 are in general less than 10^{-2} [6], so that we can expect that retaining only the terms of order s_{1a}^2, s_{2a}^2 and $s_{1a}s_{2a}$ in $\Delta\sigma$ should be an adequate approximation. Taking Eqs. (2), (3) and (5) into account, the lepton couplings needed in Eq. (8) to this approximation, for the models of interest here, are collected in Table 2.

Vector doublets	Mirror fermions	Vector singlets
$g_L^e = g_L^{e^0}$	$g_L^e = g_L^{e^0} + \frac{1}{2}s_{1L}^2$	$g_L^e = g_L^{e^0} + \frac{1}{2}s_{1L}^2$
$g_R^e = g_R^{e^0} - \frac{1}{2}s_{1R}^2$	$g_R^e = g_R^{e^0} - \frac{1}{2}s_{1R}^2$	$g_R^e = g_R^{e^0}$
$G_L^\nu = G_L^{\nu^0} - \frac{1}{2}(s_{1L} - s_{2L})^2$	$G_L^\nu = G_L^{\nu^0} - \frac{1}{2}(s_{1L}^2 + s_{2L}^2)$	$G_L^\nu = G_L^{\nu^0} - \frac{1}{2}(s_{1L}^2 + s_{2L}^2)$
$G_R^\nu = s_{1R}s_{2R}$	$G_R^\nu = s_{1R}s_{2R}$	$G_R^\nu = 0$
$G_L^N = s_{1L} - s_{2L}$	$G_L^N = -s_{2L}$	$G_L^N = -s_{2L}$
$G_R^N = s_{1R}$	$G_R^N = s_{1R}$	$G_R^N = 0$

Table 2: Lepton couplings for different lepton models in linear approximation on s_{1a}^2, s_{2a}^2 and $s_{1a}s_{2a}$. Here $g_L^{e^0} = -\frac{1}{2} + s_W^2$, $g_R^{e^0} = s_W^2$, $G_L^{\nu^0} = 1$.

The inputs in our analysis will be the following ones. We shall consider as RL or LR the simplified situations $P_1 = -P_2 = P > 0$ and $P_1 = -P_2 = -P$, respectively, with $P = 0.9$.[4] For the integrated luminosity we take $L_{int} = 20\, fb^{-1}$ as expected at the planned $0.5\,TeV$ lineal colliders [16, 19]. Concerning the experimentally accessible range for the scattering angle, we assume a $10°$ cut, i.e., $z_2 = -z_1 = 0.98$.

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[3]Actually, this reconstruction efficiency might be somewhat smaller, depending on the detector [14]. On the other hand, for our estimates we have taken a rather conservative choice for the integrated luminosity, while recent progress in machine design seems to indicate that quite larger values are attainable [16] and can compensate for the reduction of ε_W.

[4]Such a value could be attainable for electrons [18]. In the case the positron beam was unpolarized, Eq. (8) shows that one would loose a factor $1/2$ in statistics, which would affects by $\sqrt{2}$ the bounds for the cases dominated by the statistical uncertainty.

We will present the bounds on the mixing angles at the two standard deviations level (or 95% CL), which for our analysis corresponds to $\chi^2_{crit} = 4$. Also, we take $(\delta\sigma/\sigma)_{syst} = 2\%$, as currently assumed [14].

As indicated in Table 2, for the case of lepton vector doublets $\delta\sigma^{RL}$ and $\delta\sigma^{LR}$ separately constrain s^2_{1R} and $(s_{1L} - s_{2L})^2$, respectively. On the other hand, Table 2 and Eq. (10) show that that it is not possible to constrain s^2_{2R} in the adopted approximation for $\Delta\sigma$. Analogously, one expects a drastically reduced sensitivity to s^2_{1L} for mirror and vector singlet models, basically due to the cancellation between the g^e_L and the G^ν_L entries in Table 2.

The bounds on s^2_{1R} and $(s_{1L} - s_{2L})^2$ for vector doublet leptons are represented in Fig. 1, as functions of the heavy neutral lepton mass m_N. Since we are concentrating on indirect effects, for m_N we have assumed a minimum value of $250\,GeV$. Concerning the final W^+W^- polarizations, it turns out that the most restrictive limits (which can compete with current ones) result from the LL case. In Fig. 2 we report, for $m_N = 300\,GeV$, the bounds on s^2_{1R} and $(s_{1L} - s_{2L})^2$ from Fig. 1 along with the area allowed by the cross sections for RR and LL initial polarizations with, respectively, $P_1 = P_2 = \pm 0.9$ (and longitudinal W's). As a curiosity, if a "signal" (namely, a deviation from the SM) was observed for $RR + LL$ initial polarizations, separate determinations of s^2_{1R}, $(s_{1L} - s_{2L})^2$ and m_N would be possible from the combination of the three cases RL, LR and $RR+LL$. Conversely, if no signal at all was observed, still consideration of the cross section for initial $RR+LL$ might be useful to improve the limits allowed by RL and LR, for not too heavy m_N ($m_N < 130\,GeV$ for the inputs assumed here), because in this case the $RR+LL$ curve would be able to cross the region allowed by the other two cross sections. For larger m_N the limits on s^2_{1R} and $(s_{1L} - s_{2L})^2$ are determined by solely the RL and LR cross sections. In Fig. 1 we show also the bounds for the limiting case $m_N \to \infty$. Indeed, in general the most stringent constraints are obtained for larger values of m_N. This feature reflects the decreasing behaviour of the heavy neutrino exchange contribution to $\Delta\sigma$, which is proportional to r_N and opposite in sign to the other mixing effects (see Eq. (10) and Table 2). This leads to larger $\Delta\sigma$ in Eq. (16) and correspondingly to a better sensitivity on the mixing angles. In any case, the limits cannot go below those corresponding to $m_N \to \infty$ if the luminosity is fixed at the chosen value.

Turning to the other exotic lepton models, from Table 2 one can notice the following relations for the deviations from the SM:

$$
\begin{aligned}
\Delta\sigma^{LR}(\text{mirror}) &= \Delta\sigma^{LR}(\text{singlet}), \\
\Delta\sigma^{RL}(\text{doublet}) &= \Delta\sigma^{RL}(\text{mirror}), \\
\Delta\sigma^{RL}(\text{singlet}) &= 0.
\end{aligned}
\tag{17}
$$

Due to Eq. (17), in the case of no observed signal only three kinds of bounds can be derived, instead of the six ones which could be expected a priori. Specifically, in addition to the two ones for the vector doublet model exhibited in Figs. 1 and 2, the third one results from the initial LR polarization and is relevant either to the mirror

and the singlet leptons. This is represented in Fig. 3 which, shows the corresponding upper bound on s_{2L}^2 for these models.

We now discuss the case where there is Z-Z' mixing only, and no lepton mixing. In this regard, we recall from Eq. (4) that

$$\tan^2 \phi = \frac{M_Z^2 - M_1^2}{M_2^2 - M_Z^2}, \tag{18}$$

with $M_Z^2 = M_W^2/\cos^2 \theta_W$, and, since we are interested in indirect effects, $M_1 \ll E_{CM} \ll M_2$. In this case the deviations from the SM arise from the fermion coupling constants in Eq. (7) and from the Z_1 exchange amplitude which involves the Z mass shift $\Delta M = M_Z - M_1$ induced by the Z-Z' mixing angle ϕ [11]. Current limits on ϕ and ΔM from LEP are, in general, $|\phi| < 0.01$ [1]-[7] and $\Delta M < 90\,MeV$ [7].

In Fig. 4 we report the limits on ϕ as a function of the E_6 model parameter $\cos \beta$, resulting from the cross section for both longitudinal final W (which turns out to be the most sensitive one to such mixing effects), with the different initial beams polarizations. This figure shows the complementary roles of the RL and LR cross sections in limiting Z-Z' mixing over the full range of β. The typical limits on $|\phi|$ are of the order of 1.5×10^{-3} to 2.5×10^{-3}, and for selected values of β can considerably improve the present situation [1]-[7]. The orders of magnitude in Fig. 4 could be directly guessed by considering that the departure from the SM cross section relevant, e.g., to $e_L^+ e_R^- \to W_L^+ W_L^-$ entering in Eq.(16) is: $\Delta\sigma_{LL}^{RL} \propto \Delta F_1^{RL}$. In turn, ΔF_1^{RL} can be written, up to linear terms in ϕ and ΔM, as

$$\Delta F_1^{RL} = -4 \left(1 - \cot \theta_W \, g_R^e \, \chi_Z \right) \cot \theta_W \, g_R^e \, \chi_Z \left(\varepsilon_{mix} + \varepsilon_{int} + \varepsilon_{\Delta M}\right), \tag{19}$$

where $\varepsilon_{\Delta M} = -2M_Z\Delta M/(s - M_Z^2) = -7.5 \times 10^{-4}\,(\Delta M/GeV)$, $\varepsilon_{mix} = \phi(g_R'^e/g_R^e)$ and $\varepsilon_{int} = -\varepsilon_{mix}(\chi_2/\chi_Z)$. The condition $\chi^2 \leq \chi_{crit}^2 = 4$ gives the bound

$$|\phi| < \frac{1}{2}\sqrt{\chi_{crit}^2} \left(\frac{\delta\sigma_{SM}}{\sigma_{SM}}\right) \left|\frac{1 - \cot \theta_W \, g_R^e \, \chi_Z}{\cot \theta_W \, g_R'^e \, \chi_Z}\right|. \tag{20}$$

Eq. (20) gives, for example in the case of the χ-model, $|\phi| < 3 \times 10^{-3}$ with weak dependence on $\Delta M \sim 90\,MeV$. Analogously, for LR case one can simply find $|\phi| < 1.5 \times 10^{-3}$.

Finally, we discuss the case where both lepton mixing and Z-Z' mixing occur, so that the leptonic coupling constants are as in Eq. (7) and the Z_1, Z_2 couplings to W are as in Eq. (10). In this case we can look for the regions allowed to the combinations (s_{1R}^2, ϕ) and $((s_{1L} - s_{2L})^2, \phi)$, for fixed m_N. Figs. 5 and 6 show, as typical examples, the results of this analysis for the ψ-model and the χ-model, respectively, with fixed $m_N = 300\,GeV$. As one can see, the shapes of the allowed regions for the mixing angles are quite different for these two cases. From the explicit calculation it turns out that this is due to the different relative signs (depending on the value of β) between lepton and Z-Z' mixing contributions to the deviations $\Delta\sigma$. Specifically, for the ψ-model the coefficient of the ϕ-term has same or opposite sign

with respect to the lepton mixing-term (which is always negative) for the RL and LR initial polarizations, respectively. On the contrary, for the χ-model the above signs are opposite to the lepton mixing-term for both LR and RL initial polarizations.

Concerning Fig. 5 and the corresponding analysis for the ψ-model, we should notice that the limits on the Z-Z' angle ϕ are numerically quite consistent with those previously found by assuming zero lepton mixing $s_{1R}^2 = (s_{1L} - s_{2L})^2 = 0$, and numerically improve the present findings. In turn, the bounds on the lepton mixing angles s_{1R}^2 and $(s_{1L} - s_{2L})^2$ are somewhat looser than in the case $\phi = 0$ discussed above (roughly, by a factor of two), but still numerically competitive with the current situation. Finally, we can remark that the cross sections for longitudinal W^+W^- production provide by themselves the most stringent constraints for this model.

Different from the ψ-model, Fig. 6 shows that the cross sections for longitudinal W-pair production must be supplemented by the measurement of the TL final state in order to obtain the best limits for the χ-model. Also, for such model, in the range of negative ϕ the limit on Z-Z' mixing is consistent with the one obtained in the case of zero lepton mixing, while the bound somewhat worsens for positive ϕ. However, these limits can be still considered as an improvement over the present bounds. On the other hand, the constraints on the lepton mixing angles are looser than those found in the case of zero Z-Z' mixing. In this regard one should notice that there is a strong correlation between lepton mixing and Z-Z' mixing which could be very useful in order to test the χ-model. Also, it turns out that the shapes of the RL and LR allowed regions in Fig. 6 change with the CM energy, giving different intersections among the relevant curves. Thus, the combination of measurements at different energies should considerably help to reduce of allowed regions.

The examples discussed above show the advantage of combining observables corresponding to the various initial and final polarizations in order to test physical effects beyond the Standard Model which occur through deviations from the SM cross sections in process (1). As a small final remark, we would mention that the procedure presented above could be applied more extensively. A case where a similar analysis might prove useful could be the strongly interacting electroweak symmetry breaking model BESS [20]-[22]. The non-standard parameters of this model, in its simplest version, are the mass M_V of a new heavy neutral boson, its gauge coupling g'', and its direct coupling to fermions b. As an example Fig. 7 shows the bounds, which might be obtained from the combination of LR and RL cross sections for longitudinal W, on ratio of gauge couplings g/g'' and b. Also in this case, initial state polarization is found to help in reducing the region allowed to the model parameters.

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Figure captions

Fig.1 Upper bounds (95% C.L.) on s_{1R}^2 and $(s_{1L}-s_{2L})^2$ for vector doublet leptons vs. m_N, from $e^-e^+ \rightarrow W_L^- W_L^+$ and initial LR and RL polarization. Here: $E_{CM} = 0.5\,TeV$, $L_{int} = 20\,fb^{-1}$, $P_1 = -P_2 = 0.9$ (RL), $P_1 = -P_2 = -0.9$ (LR). The horizontal dashed (dotted) line indicate the bounds for the LR (RL) case with $m_N \rightarrow \infty$.

Fig.2 Upper bounds (95% C.L.) on s_{1R}^2 and $(s_{1L}-s_{2L})^2$ from $e^-e^+ \rightarrow W_L^- W_L^+$ with polarized RL, LR and $LL + RR$ ($P_1 = P_2 = \pm 0.9$) initial beams, respectively, and at $m_N = 300\,GeV$. All other inputs are the same as in Fig. 1.

Fig.3 Upper bounds on s_{2L}^2 vs. m_N for mirror (and singlet) leptons from $e^-e^+ \rightarrow W_L^- W_L^+$ with initial LR polarization. Same inputs as for Fig. 1. The horizontal dashed line indicates the bound for the LR case with $m_N \rightarrow \infty$.

Fig.4 Upper limits (95% C.L.) for ϕ vs. the E_6 model parameter $\cos\beta$ from longitudinal W pair production. Same inputs as for Fig. 1.

Fig.5 Allowed regions for the combinations (s_{1R}^2, ϕ) and $((s_{1L} - s_{2L})^2, \phi)$ for the ψ-model, from $e^-e^+ \rightarrow W_L^- W_L^+$ with RL and LR initial polarization, $m_N = 300\,GeV$. All other inputs as in Fig. 1.

Fig.6 Same as in Fig. 5, for the χ-model, from $e_L^- e_R^+ \rightarrow W_L^- W_L^+$, area enclosed between solid lines; from $e_R^- e_L^+ \rightarrow W_L^- W_L^+$, area enclosed between dashed lines; from $e_L^- e_R^+ \rightarrow W_T^- W_L^+$, area enclosed between dotted lines.

Fig.7 Allowed region (95% CL) for $(b, g/g'')$ with $M_V = 1\,TeV$, from $e^-e^+ \rightarrow W_L^- W_L^+$ with RL, LR and unpolarized initial beams.

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